Art 7 Hyperboloid of one sheet

Trace the locus of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

where a, b, c are positive and $a > b > c$

$\Rightarrow$ The equation of the surface is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \qquad \text{—— (1)}$$

where a, b, c are positive and $a > b > c$

(i) Origin :- The surface does not pass through the origin as it contains a constant term.

(ii) Symmetry :- The surface is symmetrical about the yz-plane as only as even powers of x occur.

IIly, surface is symmetrical about zx plane and xy plane.

(iii) <u>Axis - Intersection</u> :-

The surface meets x - axis where $y=0$, $z=0$

Putting $z=0$, $y=0$ in (1)

we get $\frac{x^2}{a^2} = 1$

$x = \pm a$

Surface (1) meets x - axis at $(a,0,0)$ $(-a,0,0)$

Similarly, it meets y-axis at $(0,b,0)$ $(0,-b,0)$

Again it meets z - axis where $x=0$, $y=0$

Putting $x=0$, $y=0$ in (1), we get

$-\frac{z^2}{c^2} = 1$

$z = \pm c$ i.e in Imaginary points

∴ Surface does not meet z-axis

(iv) <u>Traces on the co-ordinate plane</u> :-

Putting $z = 0$, the trace of ① on the xy-plane is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is ellipse whose major axis is along x-axis and minor axis along y-axis.

Putting $x = 0$, the trace of ① on the yz plane is $\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ which is a hyperbola whose centre is the origin and Transverse axis along the y-axis

Similarly, trace of ① on zx plane is the Hyperbola $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$

(v) <u>Generated by a variable curve</u> -

The surface meets the plane $z = k$ where

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{k^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{k^2}{c^2}$$

$\therefore$ The surface is generated by a variable ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{k^2}{c^2}, \quad z = k \quad \text{—— (2)}$$

Whose plane is parallel to XY - plane and centre (0, 0, k) moves on z - axis

Where ellipse (2) is Real whether k is positive, 0, negative.

$\therefore$ The surface lies both above and below the XY - plane

Also the semi - axes of ellipse (2) increases as k increases numerically, becomes infinite when k becomes infinite

$\therefore$ The surface extends to infinity both above and below the XY plane.

Hence shown in the figure